TILING WITH SOUND: USING RHYTHM TO CREATE SONIC TESSELLATIONS

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INTRODUCTION

WHAT IS TILING?

Tiling in geometry creates *tessellations* of repeating patterns in one or more dimensions. The analogy of tiling your bathroom floor represents the principle of tiling a two dimensional plane: a number of tiles are used repeatedly to cover a required surface area.

One of the properties of tiling is that all the available locations in the given dimension/s are accounted for, or fulfilled by the object being used to tile. Furthermore, there are no overlaps or redundant placements of the tiling object. If you re-tiled your bathroom and the contractor left one tile out, or put two on top of each other, you would not be satisfied that the job was complete!

The underlying premise of tiling, apart from creating aesthetically pleasing patterns, seems to be the elegant occupancy of the available space through the thorough repeated application of the tiling object. The isomorphic application of mathematical[1] tiling to music can be found in such fields as the 12-tone serialism (see Arnold Schoenberg and Anton Webern), tone clock theory (see Peter Schat and Jenny McLeod), canon (see J.S. Bach), total serialism generally (see Pierre Boulez and Milton Babbitt), and a specific field within it called hexachordal combinatoriality (see Milton Babbitt and Anatol Vieru's modal theory).

There are a myriad of types of tiling, even in two dimensions. Some tile the plane with small versions of the same shape (Figure 1), some with different forms of the same shape (Figure 2), and others create aperiodic patterns, extending out to cover an infinite surface area (such as the famous *Penrose tiling* of Figure 3). Johannes Kepler formalised approaches to tilings of the two-dimensional plane in his 1619 book Harmonices Mundi.[2]

Given the significant range of tiling practices and the isomorphic possibilities in music, the current study restrains the topic to focus on tiling in one dimension, as applied to musical rhythm.

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[1] Tiling in art has a companion practice in mathematics, in the areas of geometry, number theory and operator theory in functional analysis.

[2] Kepler's regular tilings comprised triangles, squares or hexagons. Paul Hindemith wrote music inspired by Kepler's writings, including his 1951 symphony *Die Harmonie der Welt*.

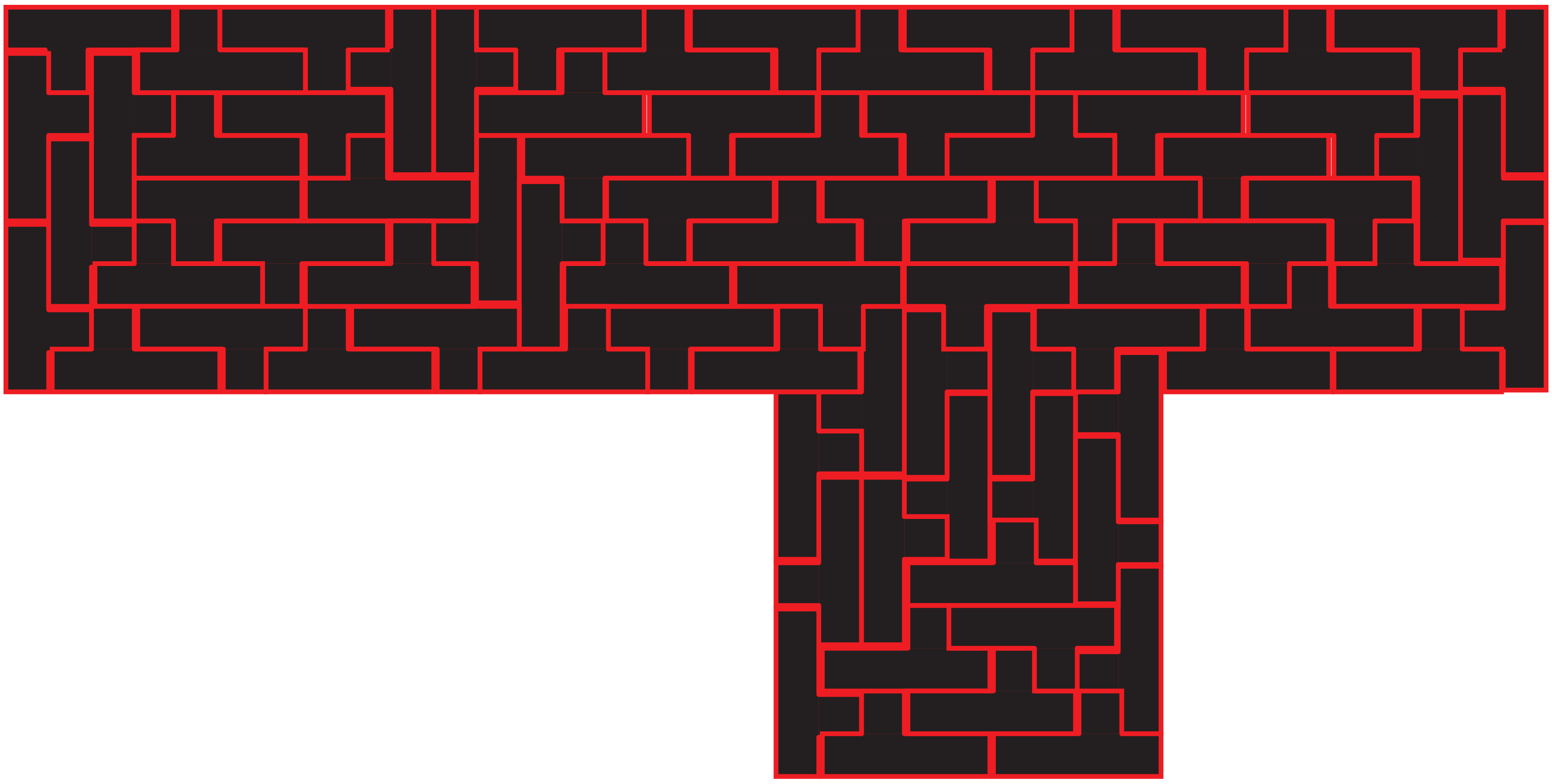

Figure 1. The “y” pentomino tiled by repeated versions of a small scale “y” pentomino.

Figure 2. The “y” pentomino tiled with the 12 versions of possible pentomino.

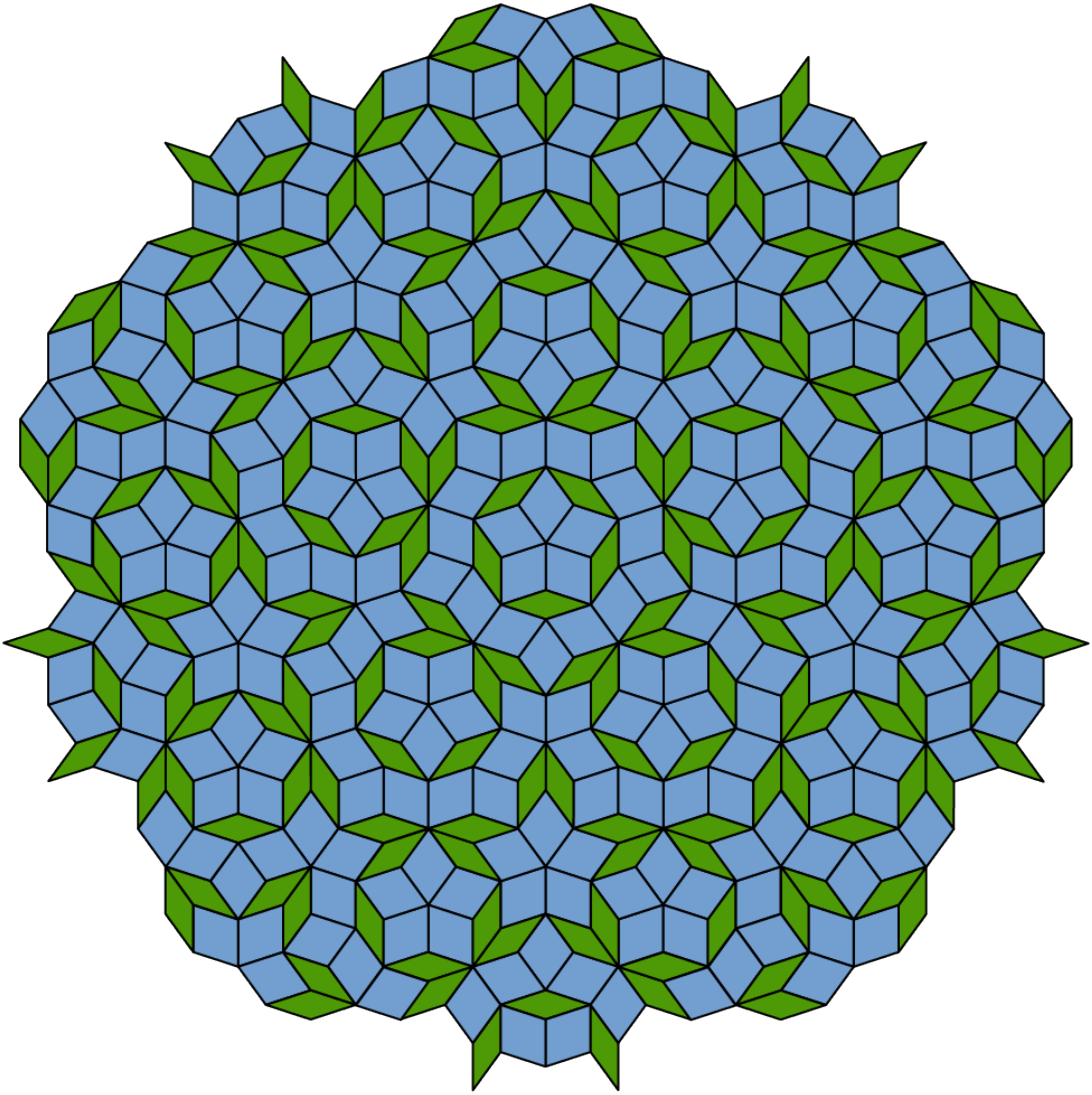

Figure 3. Penrose Tiling.[3]

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[3] Image by Geometry Guy, used under public domain provision.

TILING IN ONE DIMENSION

Simply put, a single dimension is a line, possibly with a start and an end (as in a line segment). The rhythmic analogy is intuitive, because time – being inherently chronological – has a beginning and then proceeds onward in a particular direction (Figure 4). When rhythmic schema repeat, we can take a line segment and curve the line to form a circle, which still represents one dimension, but now offers insight into such rhythmic possibilities as repetition, multiple starting points, and relationships between rhythmic activity occurring in one subsection of the circumference and another (Figure 5). Musical rhythm tends to be constrained within equidistant subdivisions of the line – receptacles for the tiling object. These are the *onsets* or attack-points of the notes that form rhythm, and are illustrated in the line segment and circular diagrams of Figures 6 and 7, respectively.

Figure 4. Visual representation of musical time as a line segment with a left-to-right chronology.

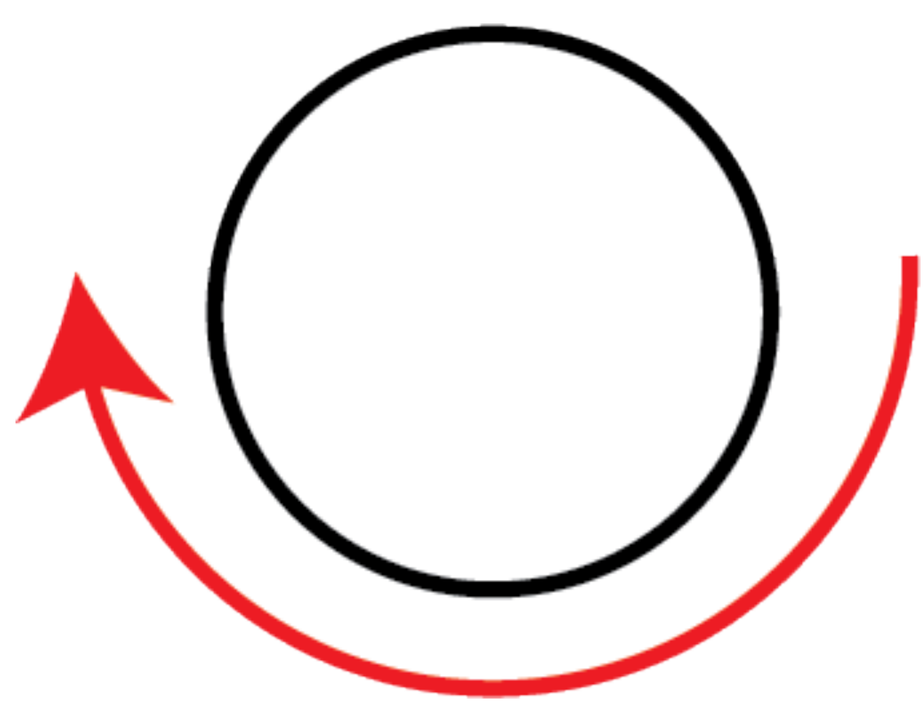

Figure 5. Visual representation of musical time as a circle with a clockwise chronology.

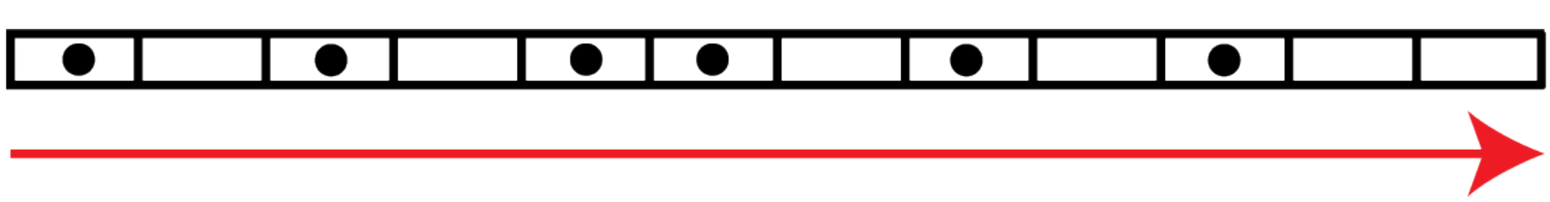

Figure 6. Visual representation of musical time as a line with a left-to-right chronology and onsets within discrete subdivisions.

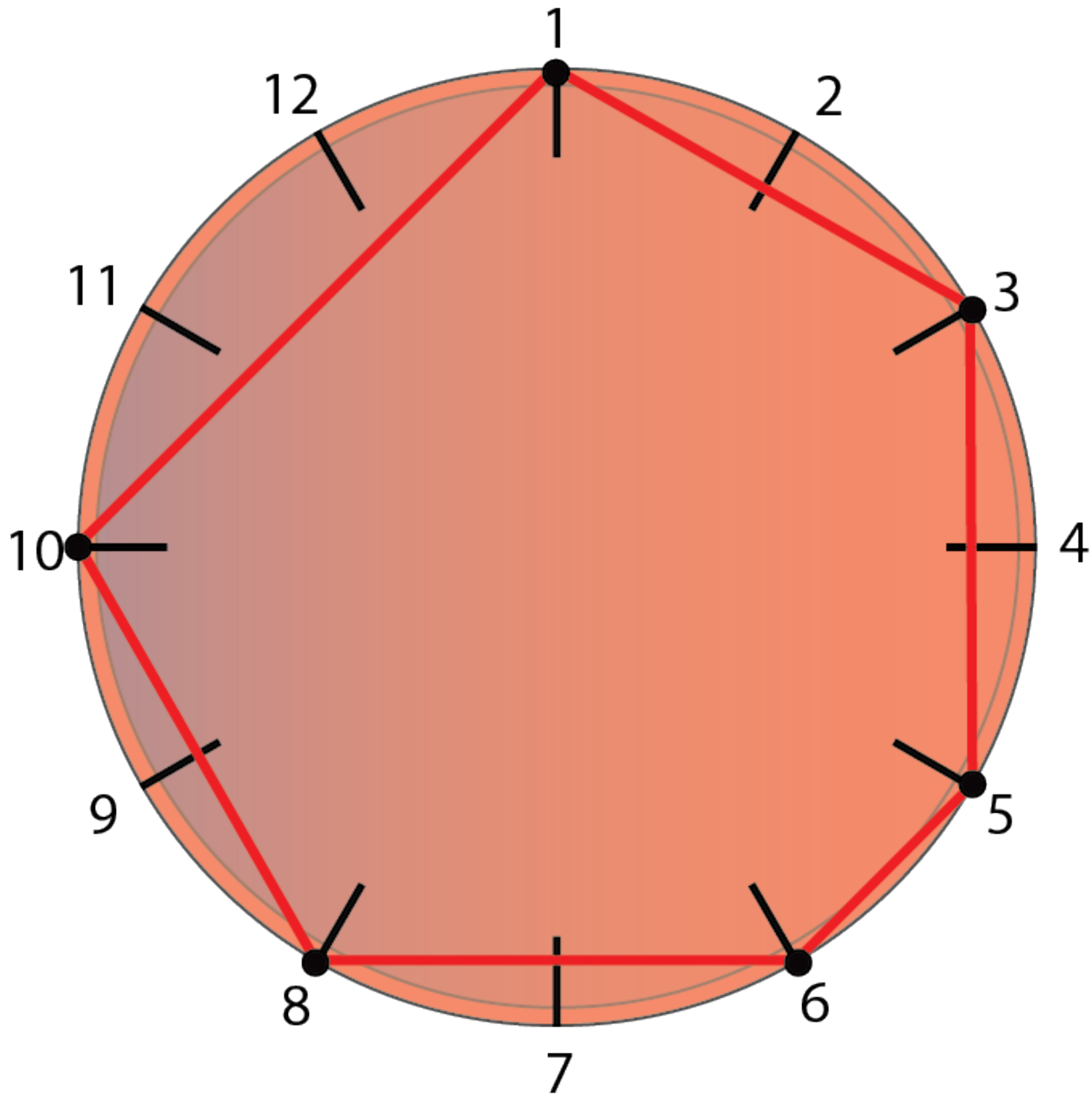

Figure 7. Visual representation of musical time as a circular "timeline" with a clockwise chronology and onsets within discrete subdivisions.[4]

[4] This is a necklace of the *fume fume* bell pattern from Ghana.

THEORY OF RHYTHMIC TILING

PULSE TRAIN

As suggested in the introduction, a bathroom floor is not considered tiled if there are gaps or overlaps. Correspondingly, the rhythm represented in Figures 6 and 7 has not tiled the time dimension because of the holes or gaps. The *pulse train* is a continuous stream of onsets, meaning that the aforementioned equidistant subdivisions of the line – the receptacles for the tiling object – are all occupied or represented. The rhythm represented in Figures 6 and 7 sounds 6 onsets out of the 12 available places for onsets.

One might wonder how a continuous stream of onsets is of any musical interest whatsoever, if it is necessary for rhythmic tiling? The answer lies in the way time is tiled, via the *motif*.

MOTIF

Simply filling all available places on a timeline with onsets might well create a pulse train but does it create a musical tiling? The answer is "maybe", but perhaps just in a trivial fashion. To explain this we have to first define what the aforementioned "tiling object" is, in the case of musical rhythm. In the case of your bathroom floor, much time and expense is dedicated towards the aesthetic and functional choice of the type and geometry of the tile used, its colour, thickness, surface contour, and even place of origin. It is the musical equivalent to the motif. The *motif*, then, is the musical idea that is repeated to create a pulse train. Whilst we are focussing on rhythm and one dimension, the motif is enriched by identifiable pitch, dynamic and timbral characteristics, so that it is memorable and able to be tracked on its journey through time.[5]

Let's start with the simplest motive of all, being a single onset which we will identify numerically as [1],[6] and a short 4-pulse timeline. Figure 8 illustrates the motif, and Figure 9 the trivial tiling in 4 voices.

Figure 8. One-onset motive [1] on a 4-pulse timeline.

[5] Even a pulse train of a single monotonous pitch can yield complex relationships when the motif applied possesses timbral variations. Toussaint likens the result to the unstable visual perception of the wireframe Necker cube (208).

[6] Johnson (2002) represents this single point motif as (0), as the first onset on his timelines is zero, not 1 as in Figure 7.

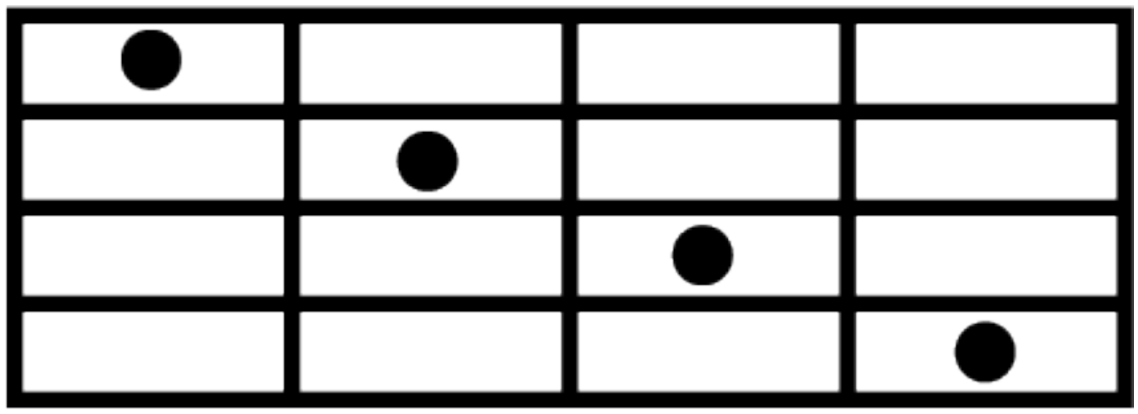

Figure 9. Trivial tiling of 4 pulses with the one-onset motif [1] using 4 voices.

Figure 9 satisfies the condition of creating a pulse train out of repeated soundings of a motif,[7] but it is not that musically interesting, even if each individual note had a beautiful and interesting timbre and pitch.[8] It's a bit like tiling 1m^2 of bathroom floor with four 25cm square white tiles!

Even simpler, but actually quite common, is the single onset two-voice tiling in Figure 10. Figure 11 interprets this as the standard rock rock beat, voiced on the bass and snare drums of the drum set.

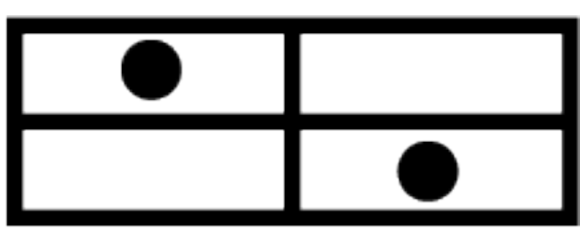

Figure 10. Trivial tiling of 2 pulses with the one-onset motif [1] using 2 voices.

Figure 11. Rock beat notation as an example of trivial tiling of 2 pulses with the one-onset motif [1] using 2 voices (played twice per bar).

Motifs begin to get a lot more interesting when they consist of a few onsets, and especially so if the onsets have a variety of adjacent and non-adjacent placements. In such situations, the different voices sounding the motif must interlock to create a pulse train, and this is

[7] This is verified by the existence of one and only one onset in each column.

[8] It is worth acknowledging that even this simple tiling places demands on an ensemble of 4 professional musicians, in that the isochronous nature of each voice's onset imparts a certain gravity, and therefore risks distortion of the equidistant pulse train.

where mathematics has been used to explore the enumerations of tilings in different categories.[9]

It is conventional that each voice renders the same motif from different points in time, but there are also musically useful situations where tiling effects are still yielded when different voices render different motives to create a pulse train.[10]

VOICES

Two or more parts or voices combine to create a tiling through the execution of the motif. The number of voices is referred to as the *order* of the tiling. Voices can be any sound source, but best results are found when they differ in pitch and/or timbre. Spatial differentiation is also beneficial.

RHYTHMIC CANON

Though canon and fugue[11] also feature successive entries of theme or motive, and are found in music since the 16th century, it was Olivier Messiaen who coined the term *rhythmic canon*, and used displacement of rhythmic motives to form his "organized chaos" of tiling rhythmic canons (Messiaen 45). Andreatta *et al* show however that even Messiaen's rigorous application of rhythmic organization does not always completely fulfil the mathematical definition of tiling, with the occurrence of the occasional gap or coincidence of voices.[12]

It was the Romanian mathematician Dan Tudor Vuza, who, inspired by the work of composer Anatol Vieru,[13] explored the topic thoroughly and invented *tessellating canons* (Norris 90). Vuza's *Regular Complementary Canons of Maximal Category* truly transferred Messiaen's *modes of limited transposition* from the pitch to rhythmic domain (Tangian 2). These canons strictly observed the tiling rule of avoiding gaps and coincidences of onset, and represent the classic geometric definition of tiling – "covering of an area by disjoint equal figures" (Tangian 1). Vuza's chief contribution was to use maths to show how to tile the time axis without inner periodicity (Andreatta 9) - namely, using factorisation of a cyclic group into two non-periodic subsets (Andreatta 11).

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[9] Examples of such tilings from musical traditions around the world will be introduced in the next section.

[10] Johnson (2002) refers to these as *complementary motifs*.

[11] The Latin derivation of the term is *fugare* and fittingly translates to mean "to chase".

[12] Andreatta *et al* investigate the tiling of Messiaen's *Harawi* and its application in other compositions (158).

[13] Vuza's model of rhythmic canons is implemented in the software Open Music, developed by IRCAM.

LINES AND LOOPS

Tiling patterns may occur on a line segment (*closed*), or some may require motive entries that eliminate the start/end boundary, best represented on a circular timeline diagram (*open*). Rather than the single onset motif of Figures 8 and 10 [1], Johnson (2002) shows that even-length motifs such as [1,3] or odd-length motifs such as [1,4] can tile *lines* or *loops* of a duration any multiple of 4 or 3 pulses, respectively.[14] Figures 12 and 13 are examples of tiling lines with motifs of even and odd lengths, respectively.

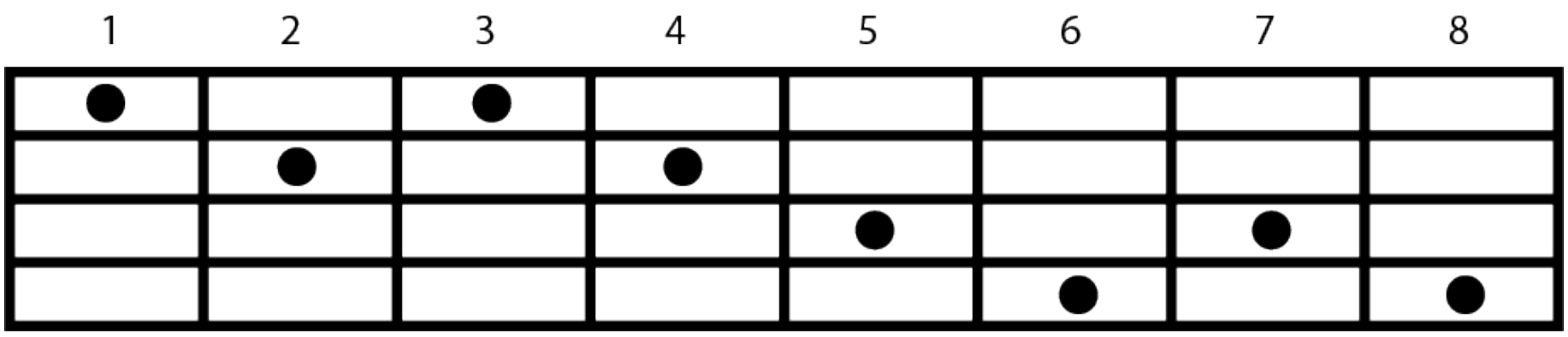

Figure 12. A tiled line of 8 pulses using the even-length motif [1,3] in four voices.

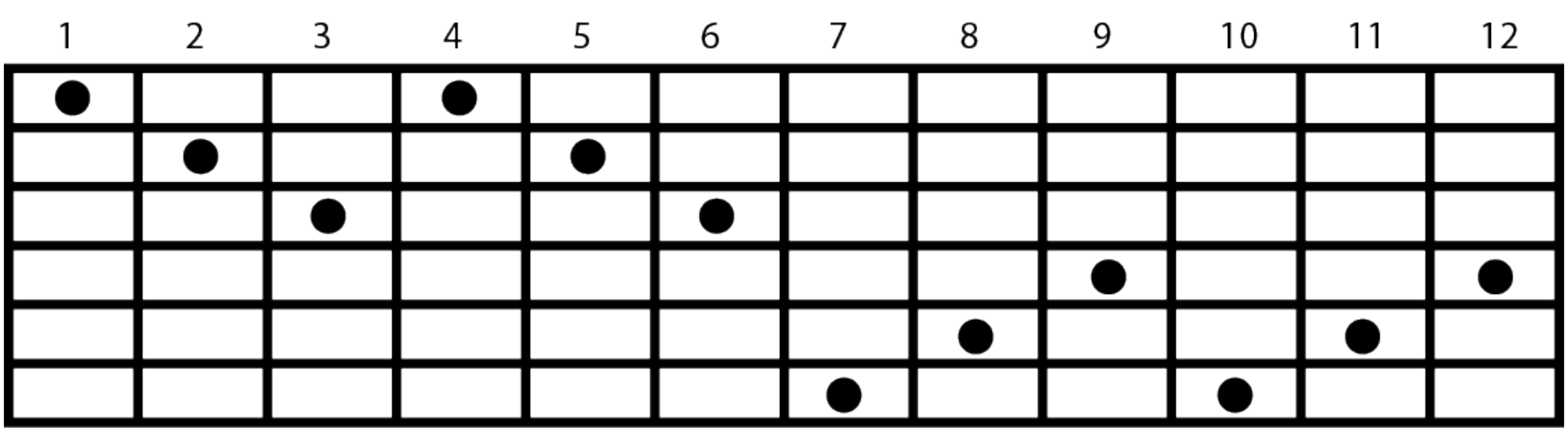

Figure 13. A tiled line of 12 pulses using the odd-length motif [1,4] in six voices.

Loops are distinguished by lines as some voices require cross-cycle relationships to maintain consistent motif durations. That is, the shortest expression of the motif for at least one voice, being the inter-onset interval between the first and last members of the motif, must cross pulse 1.[15] Loops are periodic, or cyclic.

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[14] To calculate the length of a motif, consider the integers that form the boundary attacks as the *inter-onset interval*, and subtract the first from the second numbers. [1,3] is 2 pulses long and [1,4] is 3 pulses long. This is the same procedure for calculating an interval of pitch, e.g. the C-Eb dyad is 3 semitones even though there are 4 chromatic pitches spanning the dyad.

[15] In-so-doing, calculation of inter-onset intervals requires the use of *modulo*. In the case of Figure 14, the green pair of attacks spans a distance of 3 pulses because (2-7) mod. 8 = 3.

Figure 14 illustrates a loop using motif [1,4], which in the 8-pulse cycle resembles the *baiao* rhythm of Brazil.[16] Notice that the 7-2 voice's motive (in green) crosses pulse 1.

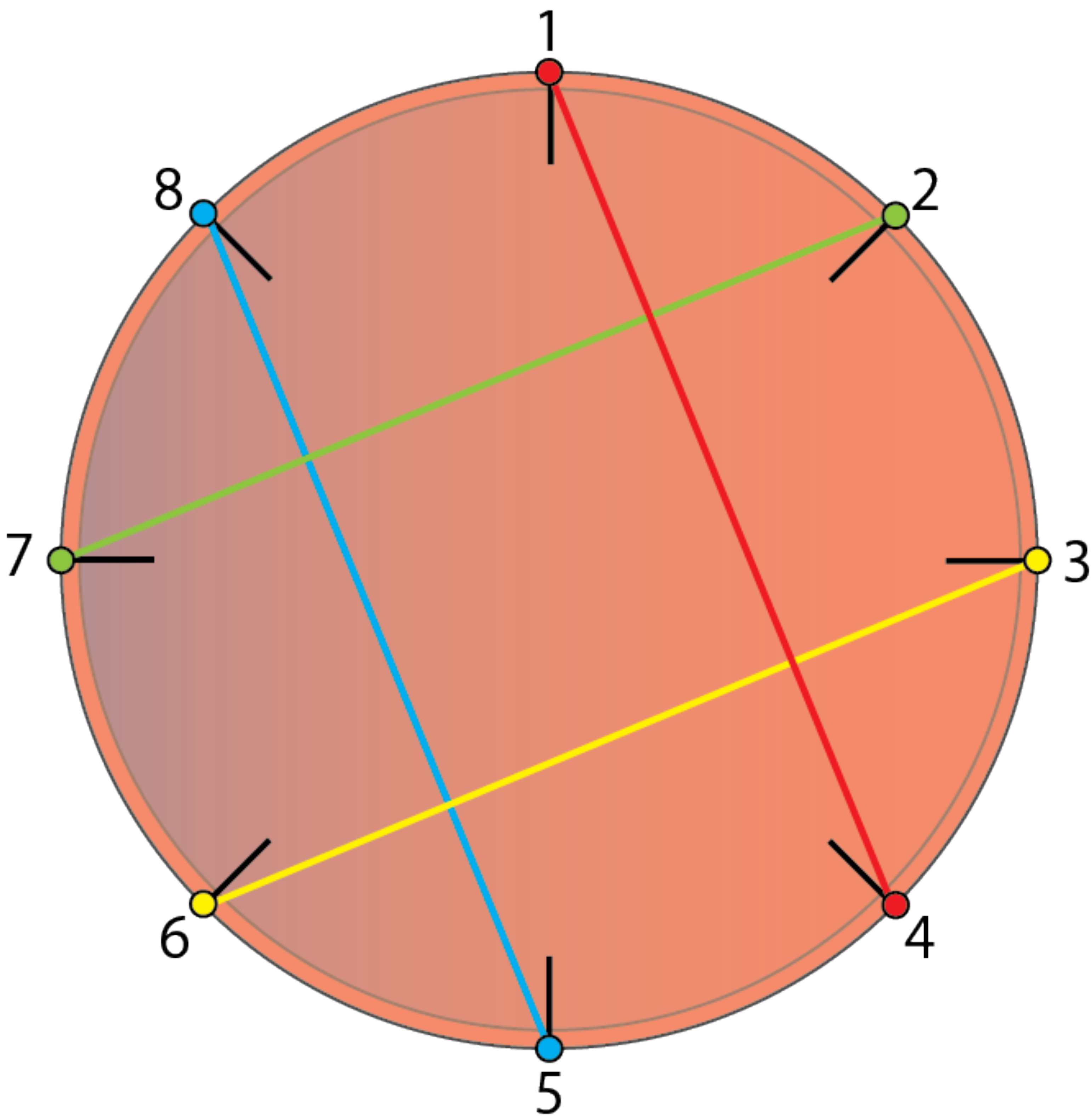

Figure 14. A tiled loop of 8 pulses using the odd-length motif [1,4] in four voices.

[16] Listeners also respond to this tiling citing other Brazilian styles including *samba* and *bossa-nova*.

In some even timelines where the (even) motif is the duration of a half cycle, the symmetry means the tiling is both a line and a loop. Figure 15 illustrates this case.

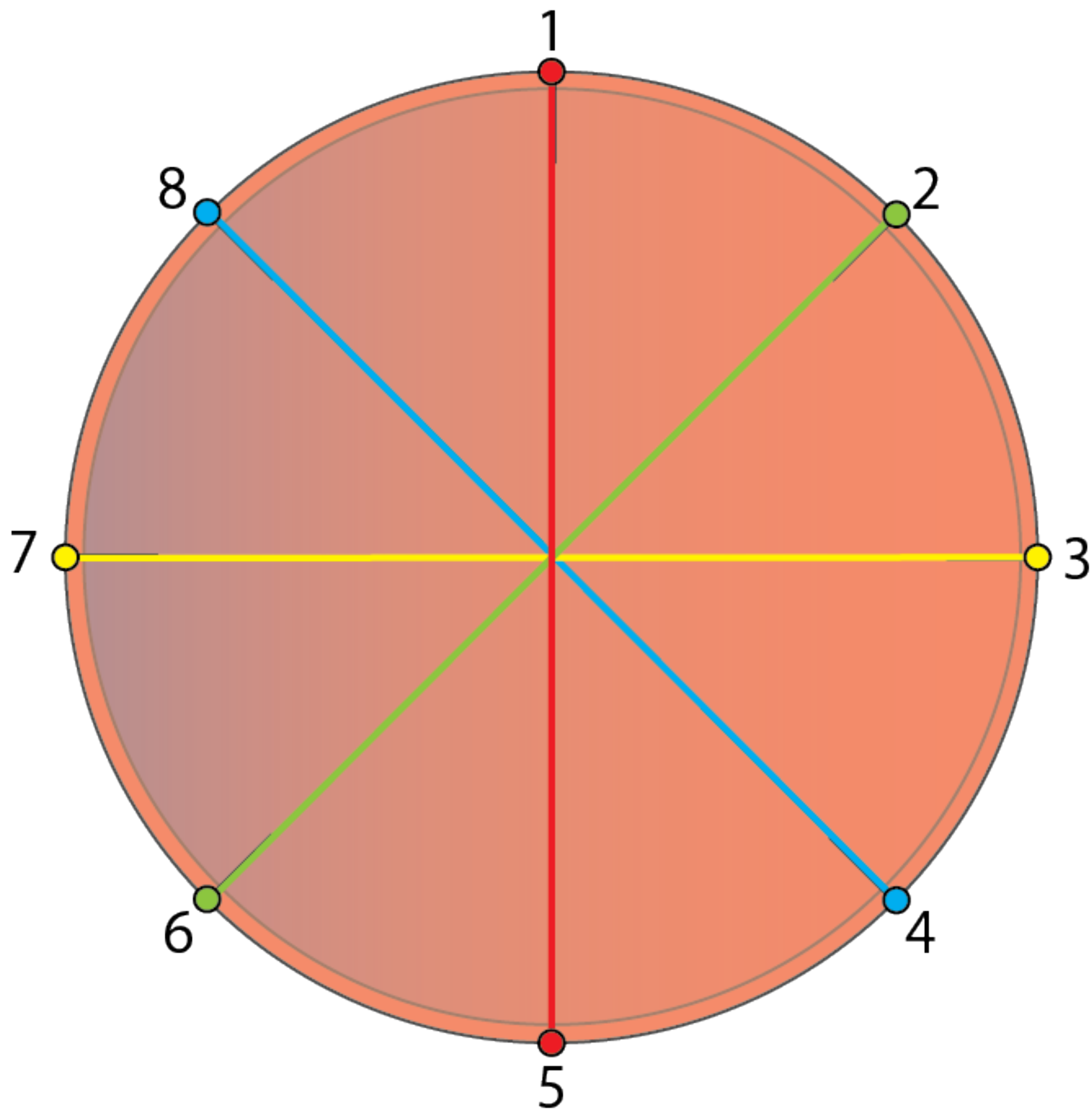

Figure 15. A tiled loop/line of 8 pulses using the even-length motif [1,5] in four voices.

Tiling loops can be considered the isomorphic equivalent of the periodicity of the 12-semitone octave.

REFLECTION RHYTHMS

A reflection possesses bilateral symmetry about an axis of reflection. In the case of an object above a horizontal axis (such as a still pond), the object is considered inverted below that axis. Figure 16 illustrates this concept.

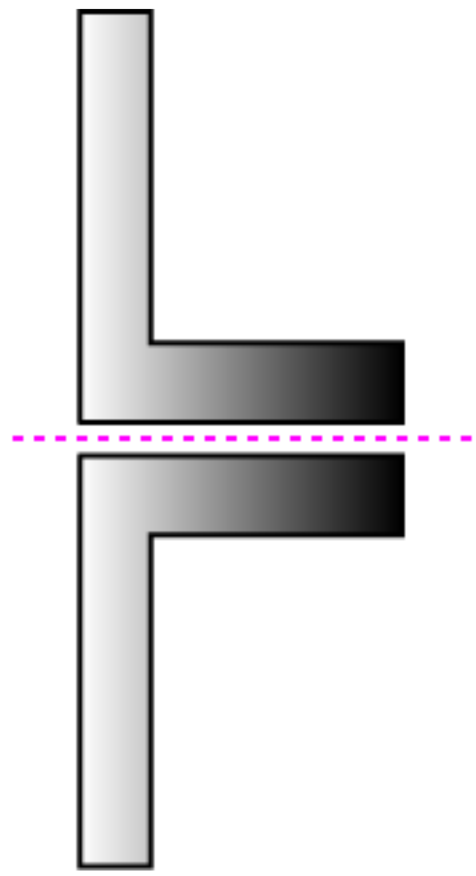

Figure 16. Reflection of the letter 'L' about a horizontal axis of symmetry.

The *complement* of a set is the collection of all the other possible items missing in that set. For instance the complement of the C major pentatonic scale is the G-flat major scale, because together the two scales complete the chromatic aggregate (of 12 semitones). In the case of rhythm, the complement consists of all those pulses in the cycle not articulated as onsets in that rhythm. Together, the original rhythm and its' complement create a pulse train.

A *reflection rhythm* is defined as one whose complement is also a mirror-symmetric reflection of the original rhythm (Toussaint 207). Refer to Figure 7 and identify those silent onsets in the cycle of 12. Figure 17 illustrates the original rhythm (red) with its complement (yellow), and provides an axis of reflection (dotted pink) which proves this example is in fact a reflection rhythm, and therefore is a tiling. Close examination will reveal this as the standard African *bell pattern*.

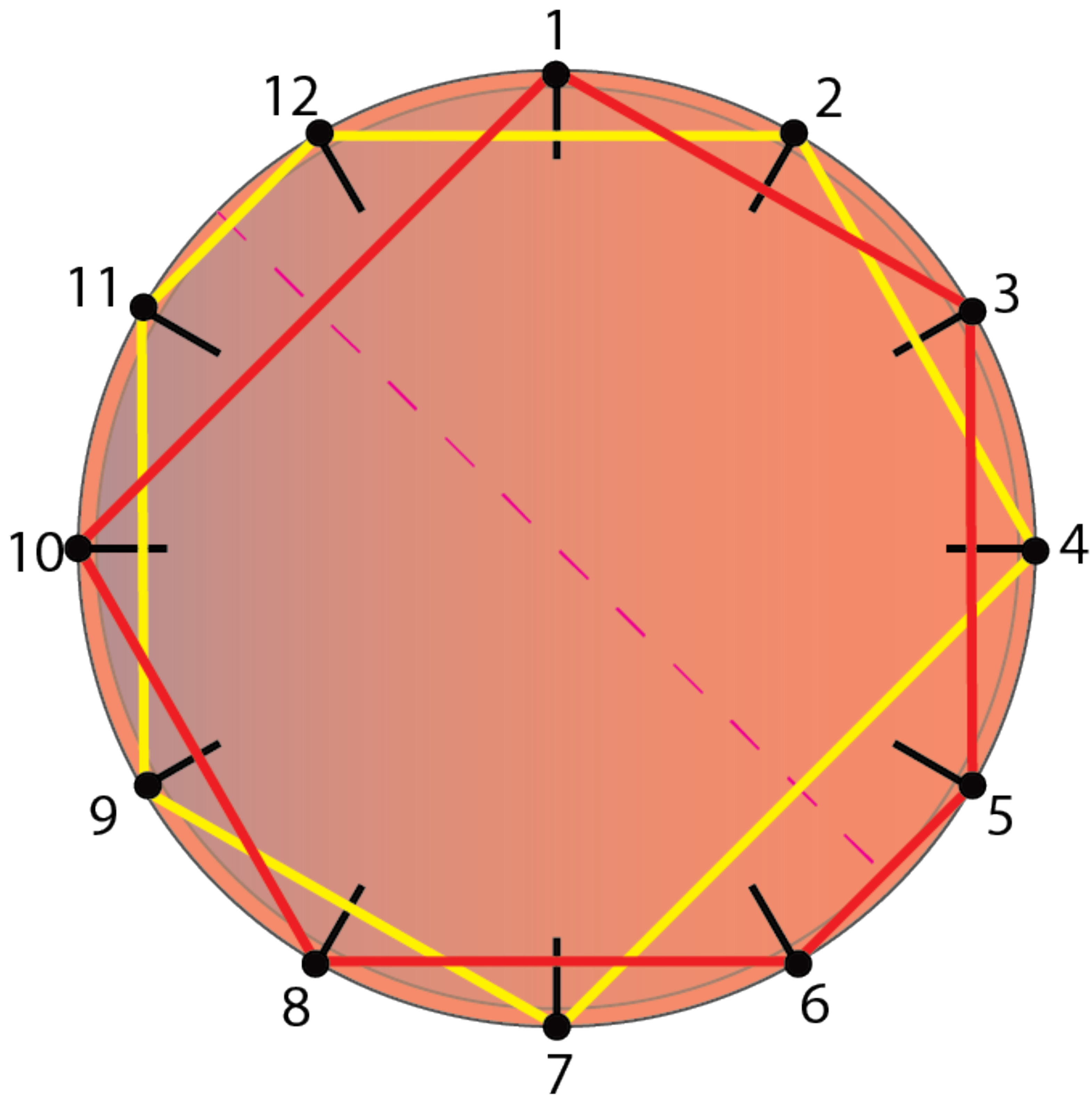

Figure 17. Reflection rhythm formed by the 6-onset rhythm of Figure 7 [1,3,5,6,8,10].

Toussaint reveals other examples of reflection rhythms common to musicians' experiences, such as the single, double and triple paradiddle from drummers' rudiments (208). As per the African *bell pattern* of Figure 17, performance of Figure 18 with different hand assignments for the two layers generates an ambiguous sensation where not only is the rhythm "bouncing" between the two hands (LRLLRLRR) but the primacy of the downbeat is diluted. The resulting ambiguity is a typical effect of reflection rhythms and tiling.

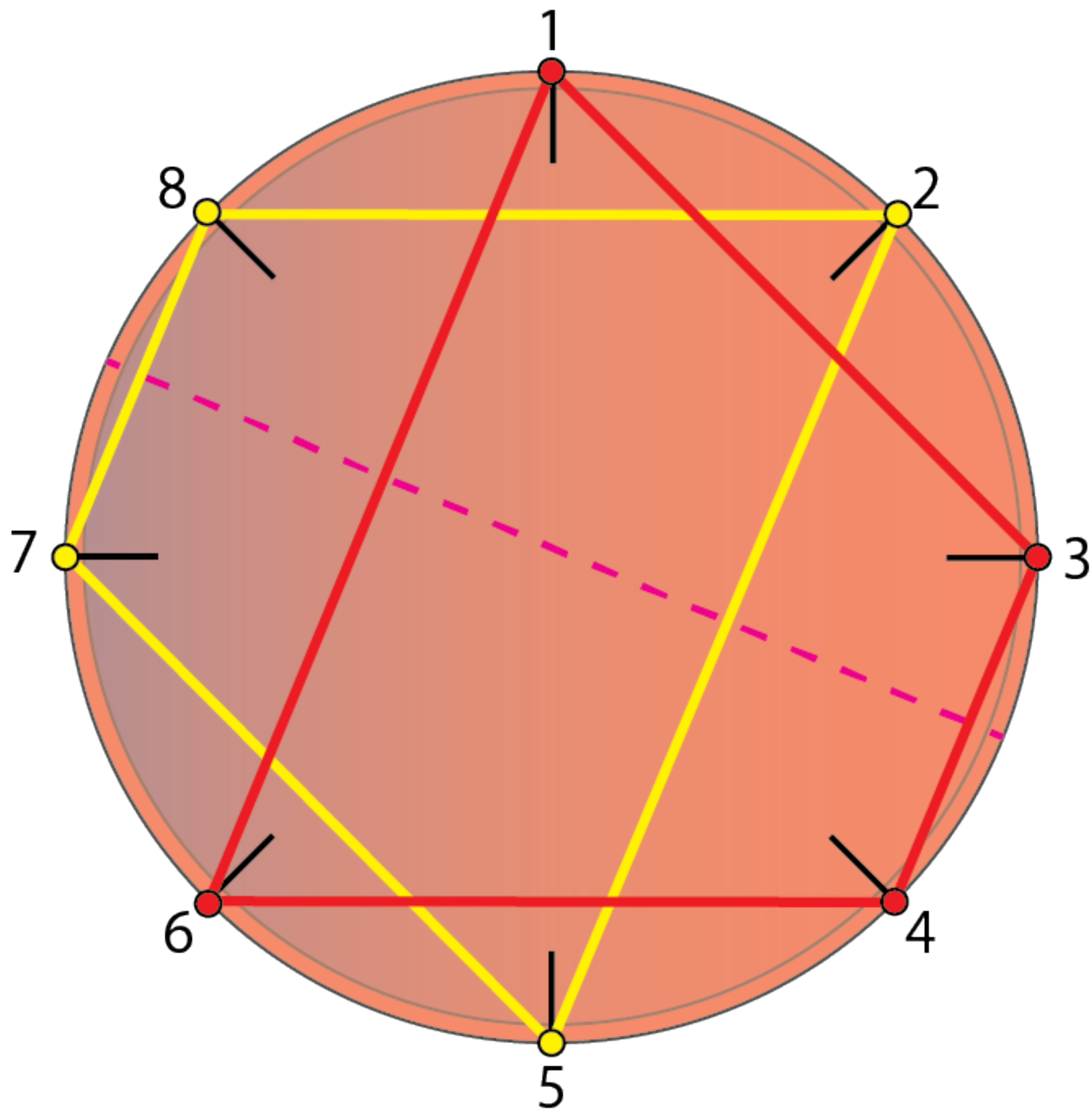

Figure 18. Reflection rhythm formed by motif [1,3,4,6] known as the single paradiddle.

In comparing the examples given in Figures 17 and 18, it can be observed that reflection rhythms require cycle lengths of an even number of pulses, in order to fulfil the bilateral symmetry requirement of two complementary motifs. They are also a loops, with the (yellow) complement commencing at the diametrically opposed pulse to pulse 1 (on pulse 7 and 5, respectively), and crossing pulse 1.

INNER AND OUTER RHYTHM

Rhythmic tiling is the process of creating a pulse stream from a motif which is translated in time and played by two or more voices. Two temporal structures are at work: the rhythm of the motif and the pattern of voice entries.[17] Different mathematicians, theorists and composers have ascribed different terms for these two types of temporal structures. In 1985 Dan Vuza determined the relevant *supplementary sets* which he designated set "R" and "S" (Andreatta 4). Figure 19 shows how this terminology correlates between different authors after Vuza.

Composer/Attribute	Vuza	Messiaen	Andreatta
Motif	Set R / ground	*pedale rhythmique*	inner rhythm
Motif entries	Set S / metric class		outer rhythm

Figure 19. Table showing equivalent terminology for tiling motives and their translation.

It is worth noting that whilst the inner rhythm defines onset numbers on the timeline (e.g. [1,3] being a two-onset motif on pulse 1 and 3), the outer rhythm identifies inter-onset intervals, or the distance of translation.

Fripertinger is another mathematician/musician who has scrutinized rhythmic tiling canons and enumerated their possibilities.[18] Figure 20 is an excerpt from one of his algorithmic lists. In Fripertinger's research he defines *weight* of the rhythm of a certain length as the number of onsets in a *function* (motif), and its *vector* as the number of members of the *function* equal to 1. The *inner* and *outer rhythms* are shown in the left and right columns, respectively.

1 canon: [00000000 11111111]	[0000000100000001]
2 canon: [00000010 11111101]	[0000000100000001]
3 canon: [00000100 11111011]	[0000000100000001]
4 canon: [00000110 11111001]	[0000000100000001]
5 canon: [00001000 11110111]	[0000000100000001]

[17] Whilst we are considering tiling in one dimension in the rhythmic domain, the same considerations have been applied to pitch in 12-tone serialism, such as in the concept of *hexachordal combinatoriality* (see Forte). Schat and McLeod's use of the tone-clock system and the *steering* of interval prime forms is another example. The mathematical principle of the algebraic features of *finite cyclic groups* underpins these applications.

[18] Refer to Fripertinger's web site. https://imsc.uni-graz.at/fripertinger/

6 canon: [00001010 11110101]	[0000000100000001]
7 canon: [00001100 11110011]	[0000000100000001]
8 canon: [00001110 11110001]	[0000000100000001]
9 canon: [00010010 11101101]	[0000000100000001]
10 canon: [00010100 11101011]	[0000000100000001]
11canon: [00010110 11101001]	[0000000100000001]
12 canon: [00011000 11100111]	[0000000100000001]
13 canon: [00011010 11100101]	[0000000100000001]
14 canon: [00100100 11011011]	[0000000100000001]
15 canon: [00101010 11010101]	[0000000100000001]
16 canon: [00101100 11010011]	[0000000100000001]

Figure 20. 16 pulse tiling canon list from Fripertinger – all possible weight 8, 2 voices.

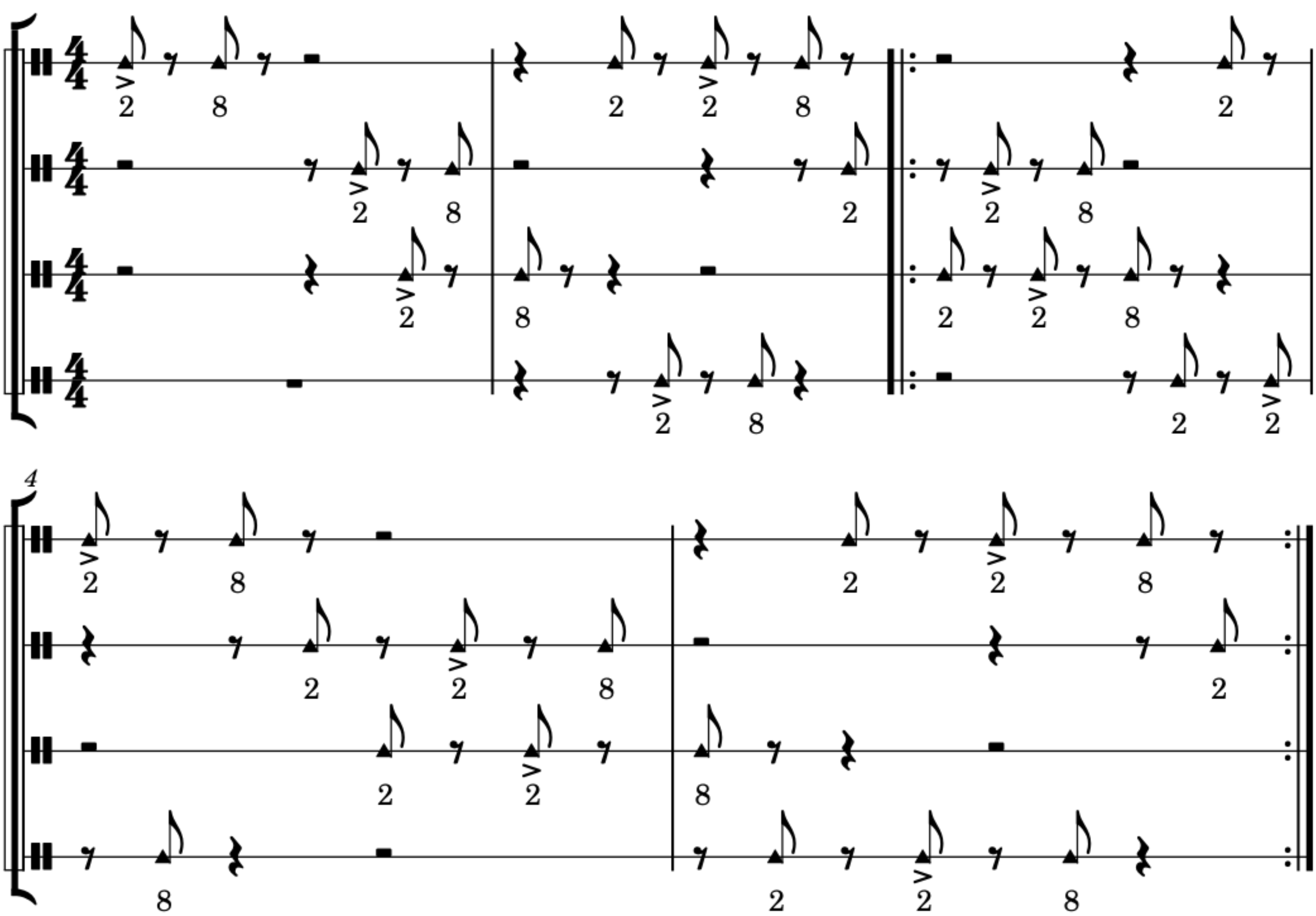

Figure 21. Tiling canon with [2,8,2] inner rhythm and (1,6,7,12) outer rhythm.

Figure 21 shows in notated form a tiling canon (loop) that applies the [2,8,2] motif (*inner rhythm*) in four voices with entries on pulses 1, 6, 7, and 12 (being the *outer rhythm*).[19]

[19] Adapted from Andreatta et al, p.158.

TAXONOMY OF TILING IN ONE DIMENSION

Lines and *Loops*
Closed and *Open* tilings were previously discussed, as being one-dimensional tilings with a beginning and an end (line/closed/aperiodic) or that wrap around (loop/open/periodic). Open tilings may require motive entries that eliminate the start/end boundary, best represented on a circular timeline diagram. Figures 22 and 23 illustrate both categories using the same [1,3] motif using notation.

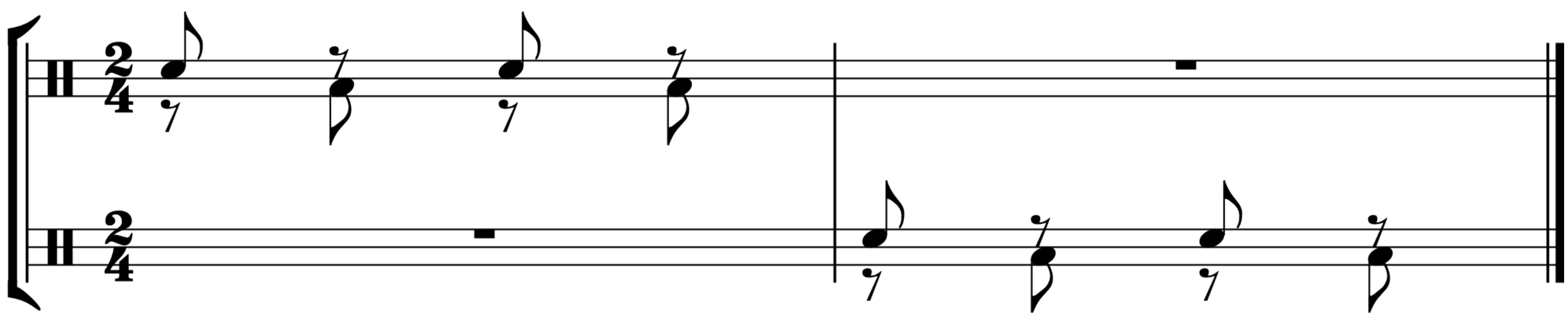

Figure 22. Closed tiling (line) of the motive [1,3] in four voices.

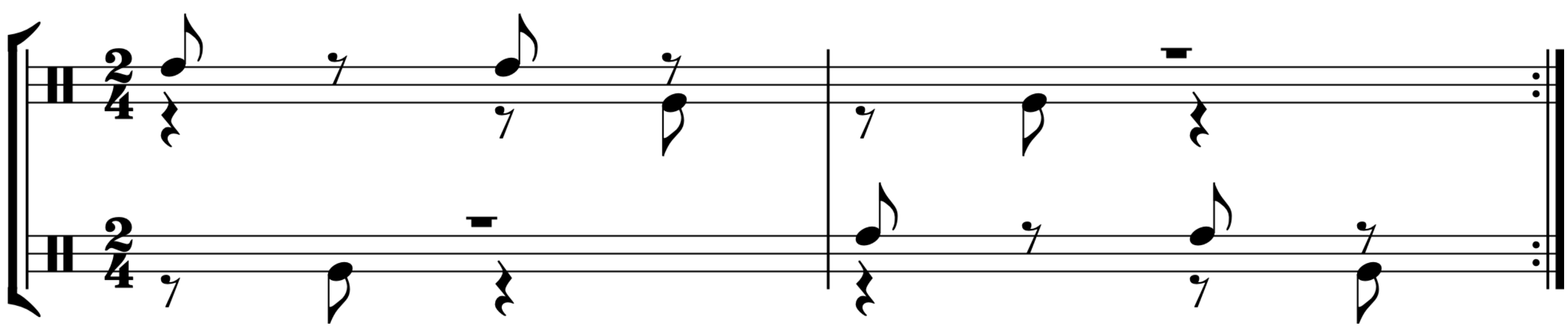

Figure 23. Open tiling (loop) of the motive [1,3] in four voices.

The musical effect of lines and loops can be quite different. The symmetry required to create a periodic tiling loop generates a feeling of stasis, harmony and rest. By contrast, a tiled line tends to feel directed, embodying motion and freedom.[20] Indeed, a tiled line has finite length and presumably is followed by other material – be it another type of tiling or material categorically different in nature.

Uniform or *non-uniform*
When a single motif is exclusively amongst the voices, it is a uniform tiling. In uniform tilings, the cycle length (denoted n) must be divisible by the number of voices (the order denoted o) and the number of onsets in the motif (denoted m). E.g. a cycle 21 pulses long with 3 voices each with a 7-onset motif. The pulse train is shared equally by all the voices.
$n = o \times m$

[20] Aesthetic concepts of symmetry are discussed by Weyl.

Figure 1 geometrically illustrates a uniform tiling, and the reflection rhythm of Figure 17 is notated in Figure 24.

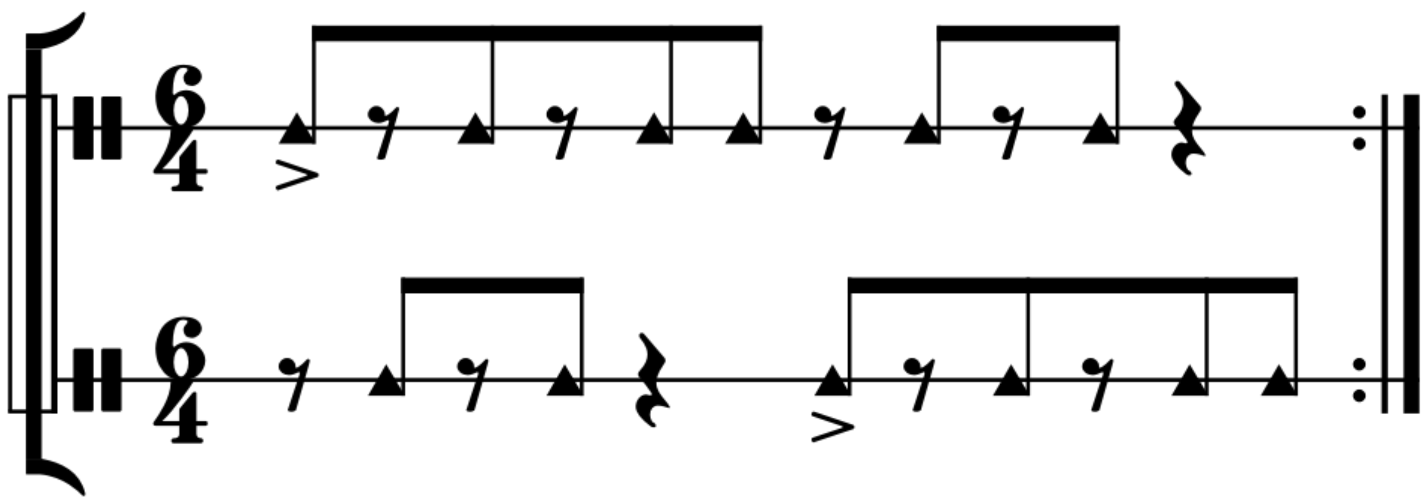

Figure 24. Uniform tiling of the motive [1,3,5,6,8,10] in two voices – double paradiddle.

Non-uniform tilings combine different motifs, possibly of different lengths. Figure 2 geometrically illustrates a non-uniform tiling.[21] Figure 25 provides a notated example. Non-uniform tilings may however be categorised also as being *perfect* if their motifs differ only by scale (see below).

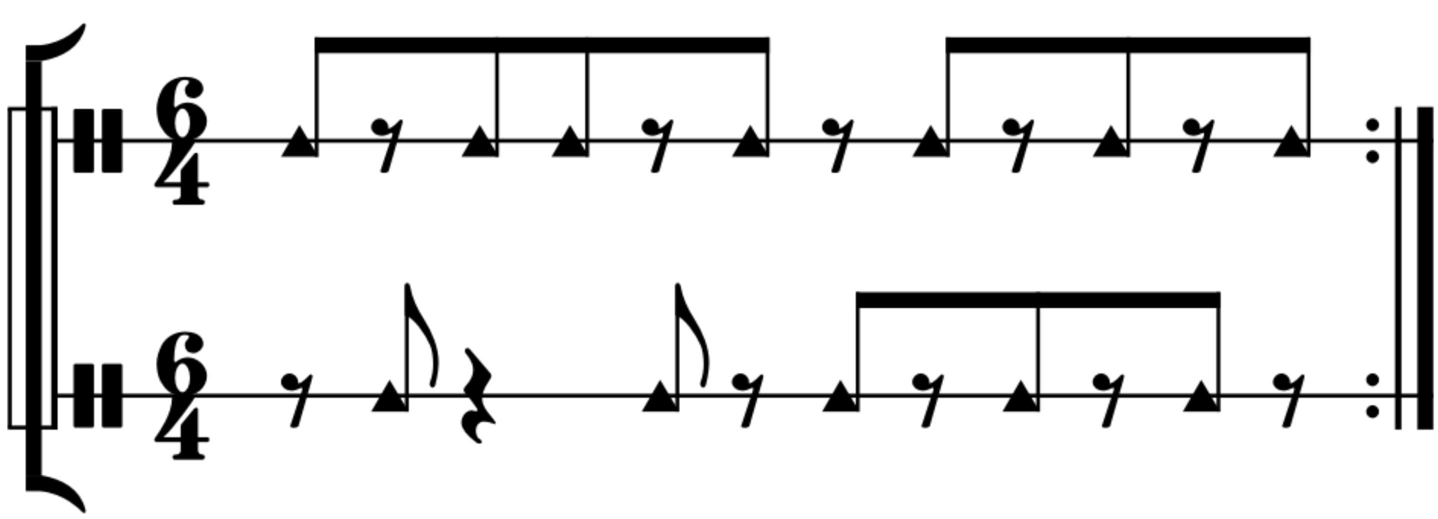

Figure 25. Non-uniform tiling in two voices.

[21] Even though all the intrinsic shapes are pentominoes – which provides congruence to the overall structure – the different forms of pentomino are different shapes and are also not proportionally related.

Trivial or *non-trivial*
Trivial tilings employ simple isochronous (beat-like) motifs such as in Figures 9 and 10, and the notated example in Figure 26. Non-trivial tilings concern all other types of motifs.

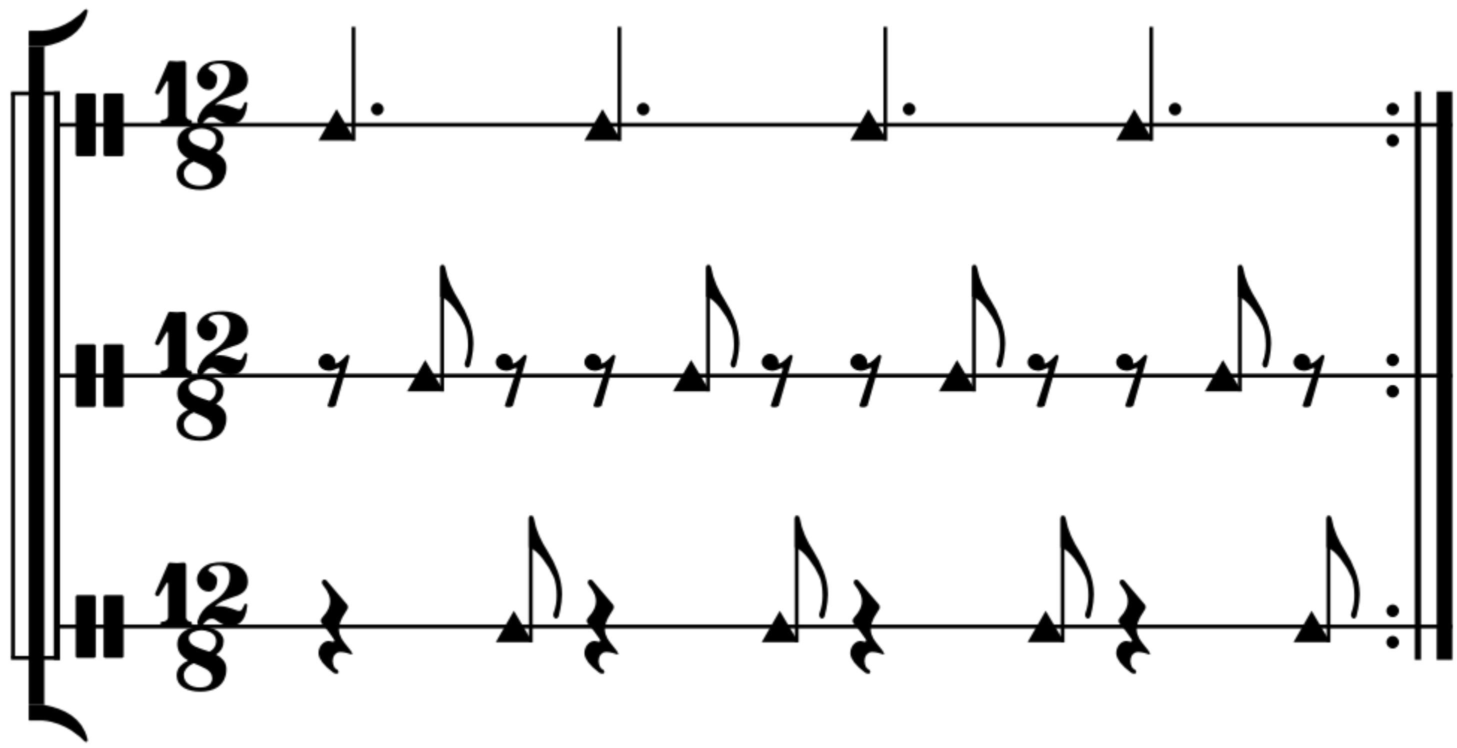

Figure 26. Trivial tiling in three voices.

Adjacent onsets or *non-adjacent onsets*
Motifs may contain only adjacent onsets, as in Figure 27, or more interestingly, contain gaps which receive onsets from disjoint motifs to create interlocking effects – interlocking reflection rhythms.[22] (Figure 21 is one notated example.)

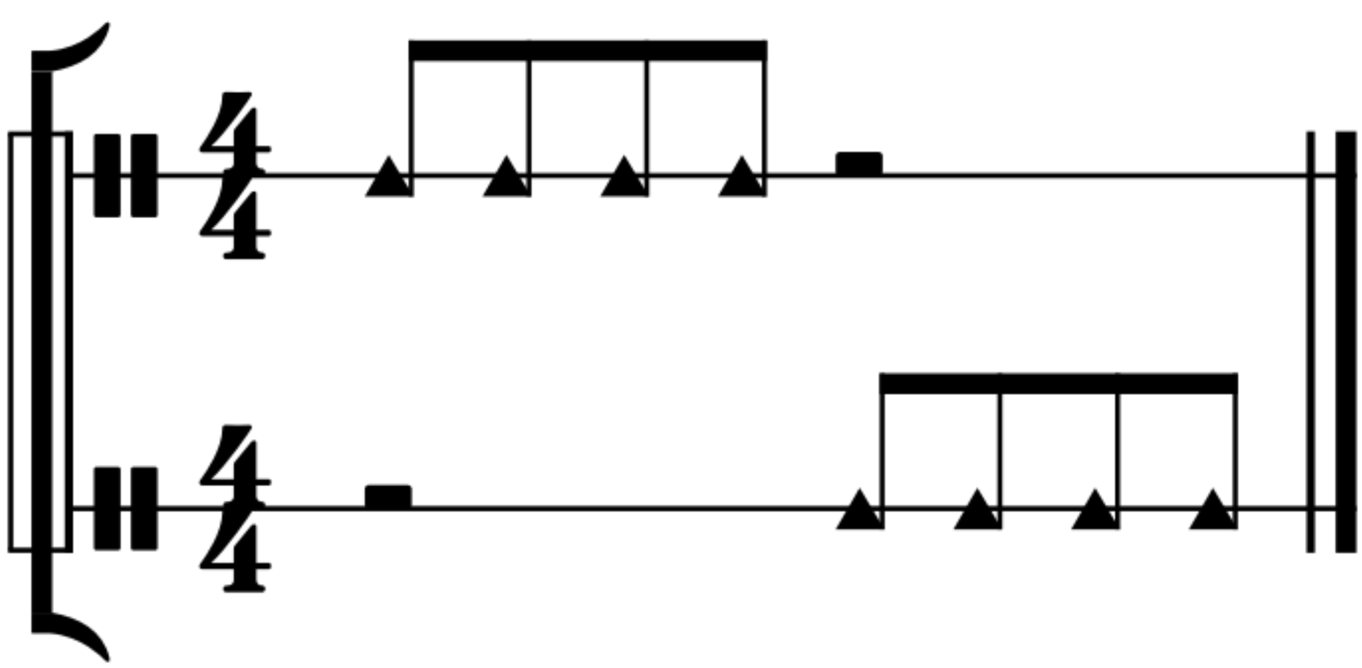

Figure 27. Adjacent onset motif [1,2,3,4] tiling in two voices.

[22] This effect has been explored through music of the ages including the *hocket* technique of medieval music.

Scaled or *un-scaled*
Sometimes a motif may appear proportionally scaled in a tiling, via rhythmic augmentation or diminution. Such rhythmic canons are sometimes categorised *Noll Canons* in observance of the contribution of Thomas Noll to their development (Agon & Andreatta 23). Scaled tilings are more generally considered *perfect tilings* (see below). Un-scaled tilings are the default and inferred category, so *Noll Canons* are an exception.

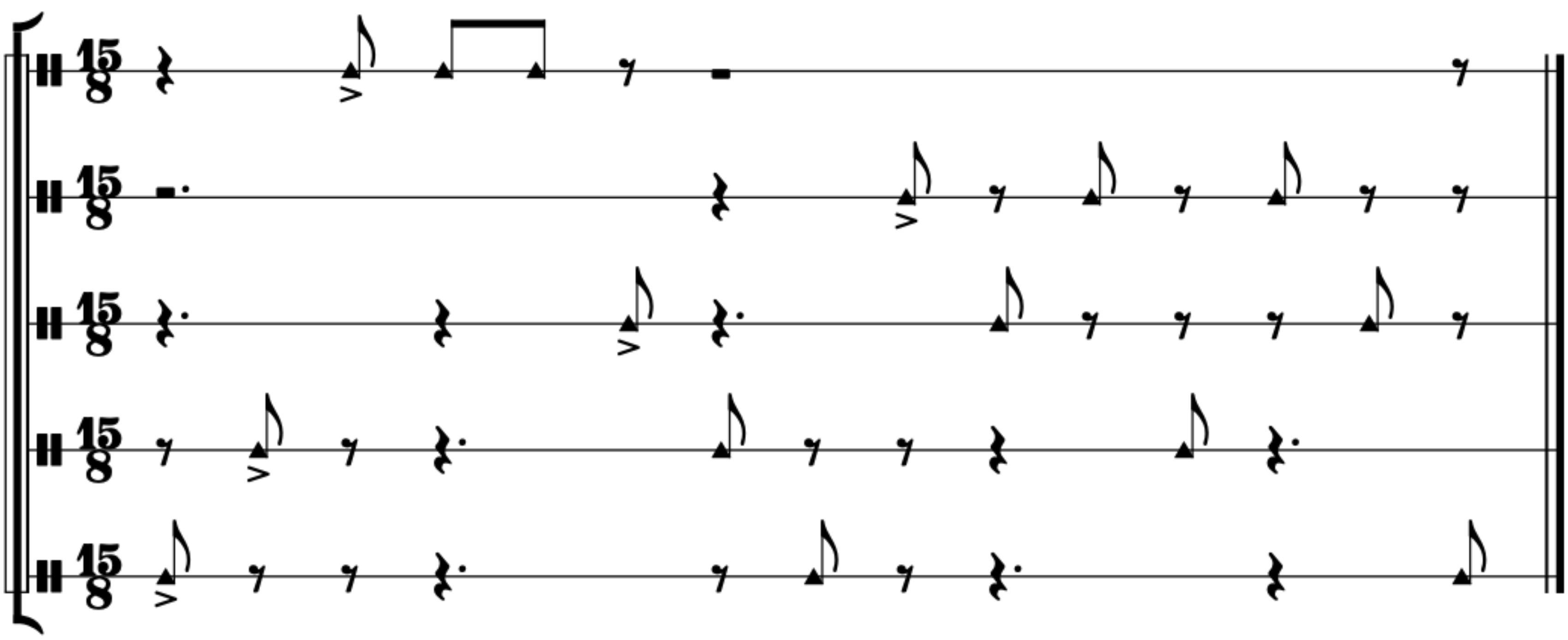

Figure 28. Scaled tiling featuring the triplet motif [1,2,3] in five voices. The four other scalings are [1,3,5], [1,5,9], [1,6,11] and [1,8,15] respectively from voice 1 to 5.[23]

Perfect or *imperfect*
Tilings that use only one motif in multiple scalings are considered perfect tilings (even though they are inherently *non-uniform*). Perfect rhythmic tilings are analogous to perfect tilings in two dimensions, where a geometric shape – such as a square – is tiled by number of smaller squares all of different dimensions. The lowest order perfect squared square was discovered by A.J.W. Duijvestijn and is illustrated in Figure 29. Johnson (2004) illustrates the rhythmic application of the associated combinatorial mathematics, such as the tiling of a line of 15 pulses in 5 voices with triplets of different scalings, notated in Figure 28.

Imperfect tilings combine different motifs that are not proportionally related, and possibly are of different lengths. They are consequently synonymous with *non-uniform* tilings, illustrated in Figure 30.

[23] Johnson (2004).

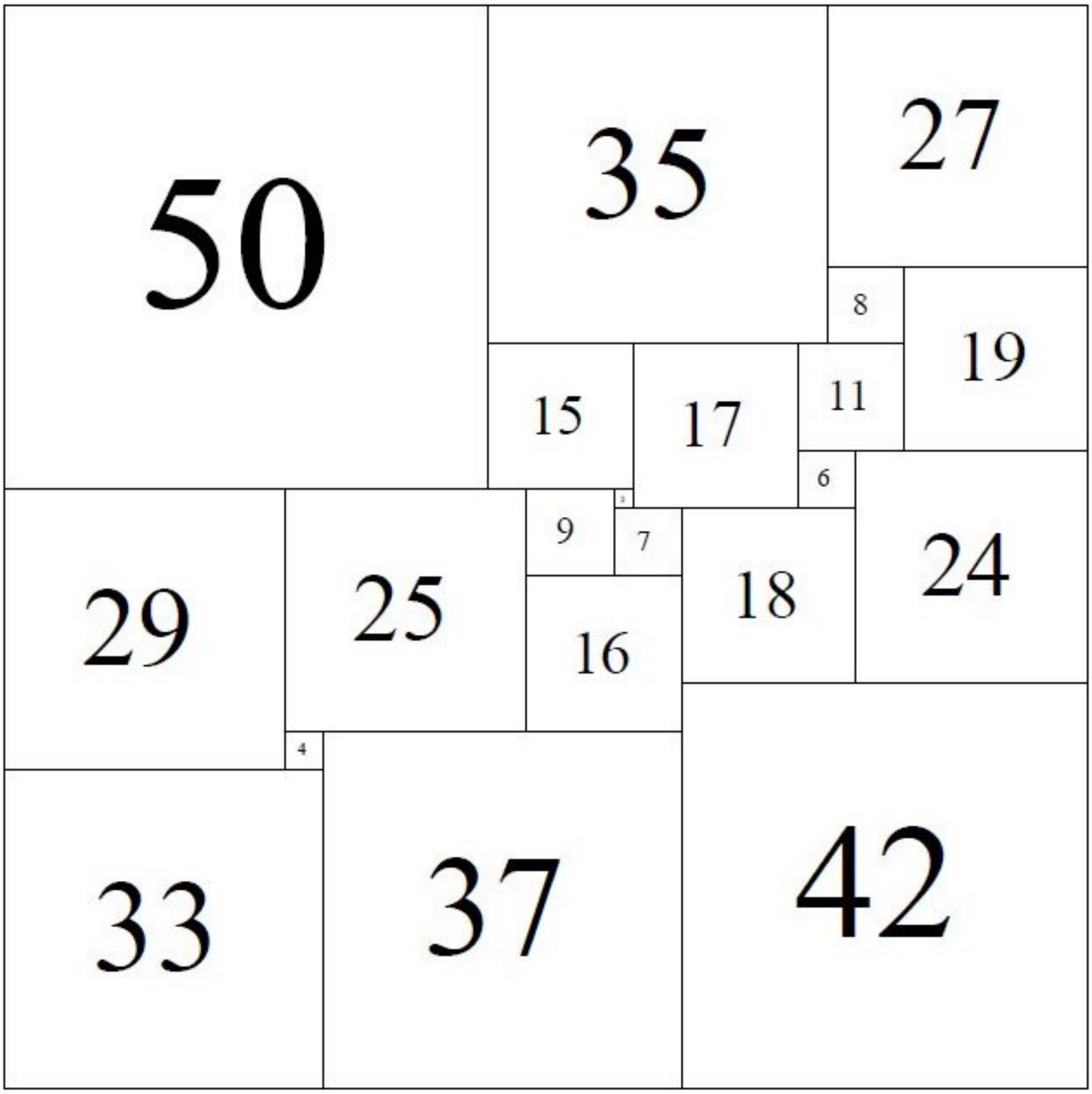

Figure 29. Geometric illustration of perfect tiling. This is the lowest order perfectly squared square discovered by Duijvestijn.

Figure 30. Imperfect tiling in three voices.

Disjoint or *Partially disjoint* motifs

Tilings by definition are formed by disjoint[24] motifs, with no overlaps. However there are times – even in Messiaen as we have seen – where musically pleasing results arise from divergences from this definition. Tilings with partially disjoint motifs may see shared onsets and/or gaps or rests in the pulse train, as in Figure 31.

[24] Disjoint sets in mathematics lack common elements.

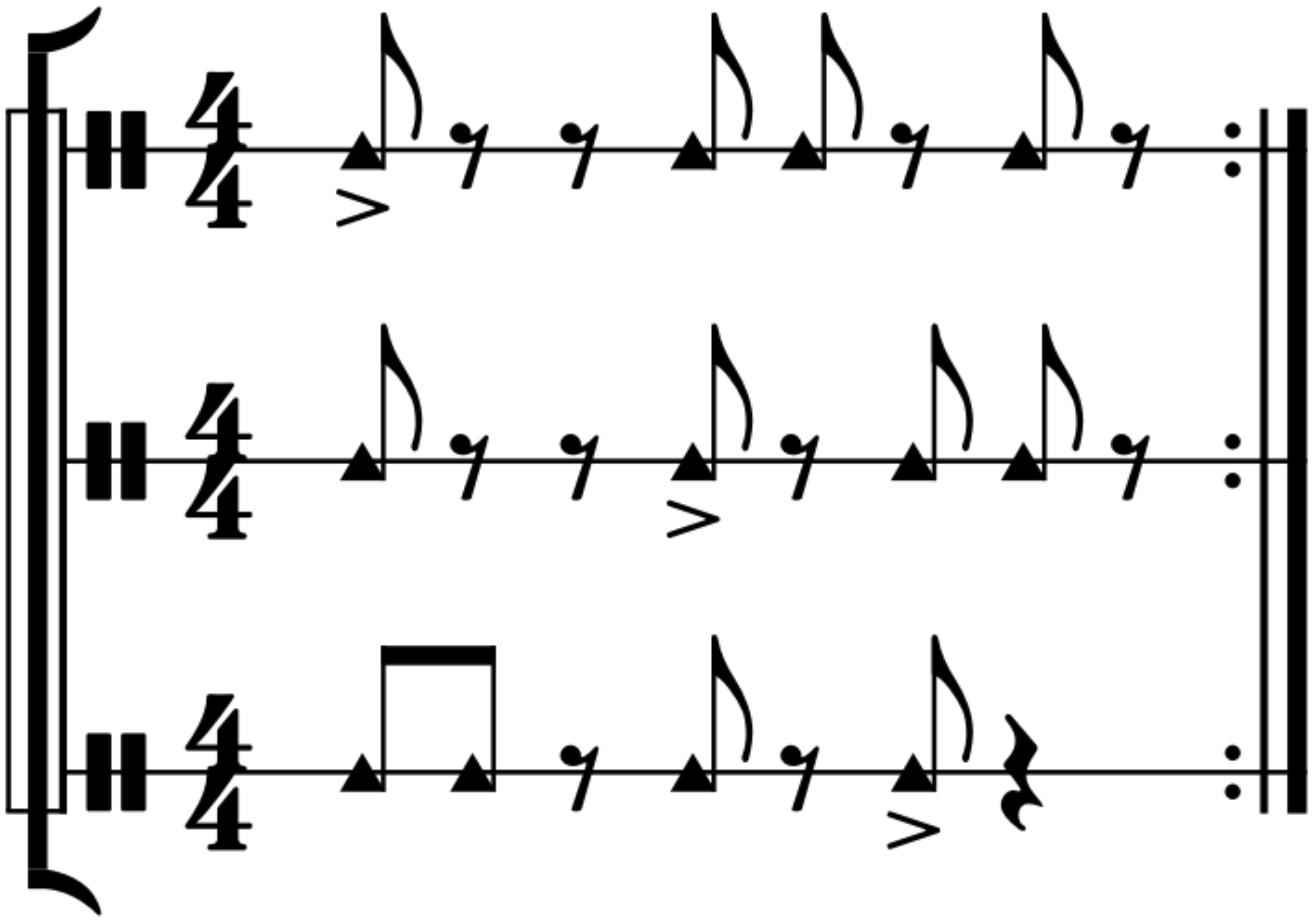

Figure 31. Partially disjoint tiling in three voices, using motif [1,3,4,6] with outer rhythm (10010100) in 8-pulse loop.

REGULAR COMPLEMENTARY CANONS OF THE MAXIMAL CATEGORY

A special category of rhythmic tiling is shown by Vuza, often referred to as the "Vuza Canons" (Andreatta 9). These tile time without any inner periodicity. That is, the motif loops the cycle in such a way as no cycle is a repetition of any other.[25] Hence, these tilings resemble the Penrose tilings mentioned in the introduction.

Vuza discovered that the shortest such canon has a pulse length of 72.[26] Figure 32 illustrates such a canon.[27]

[25] See Fripertinger (2002) for the mathematical proofs of such special tilings.

[26] See Fripertinger (2002) for a table of results of the *Vuza constructible canon* algorithm.

[27] Adapted from Andreatta (10).

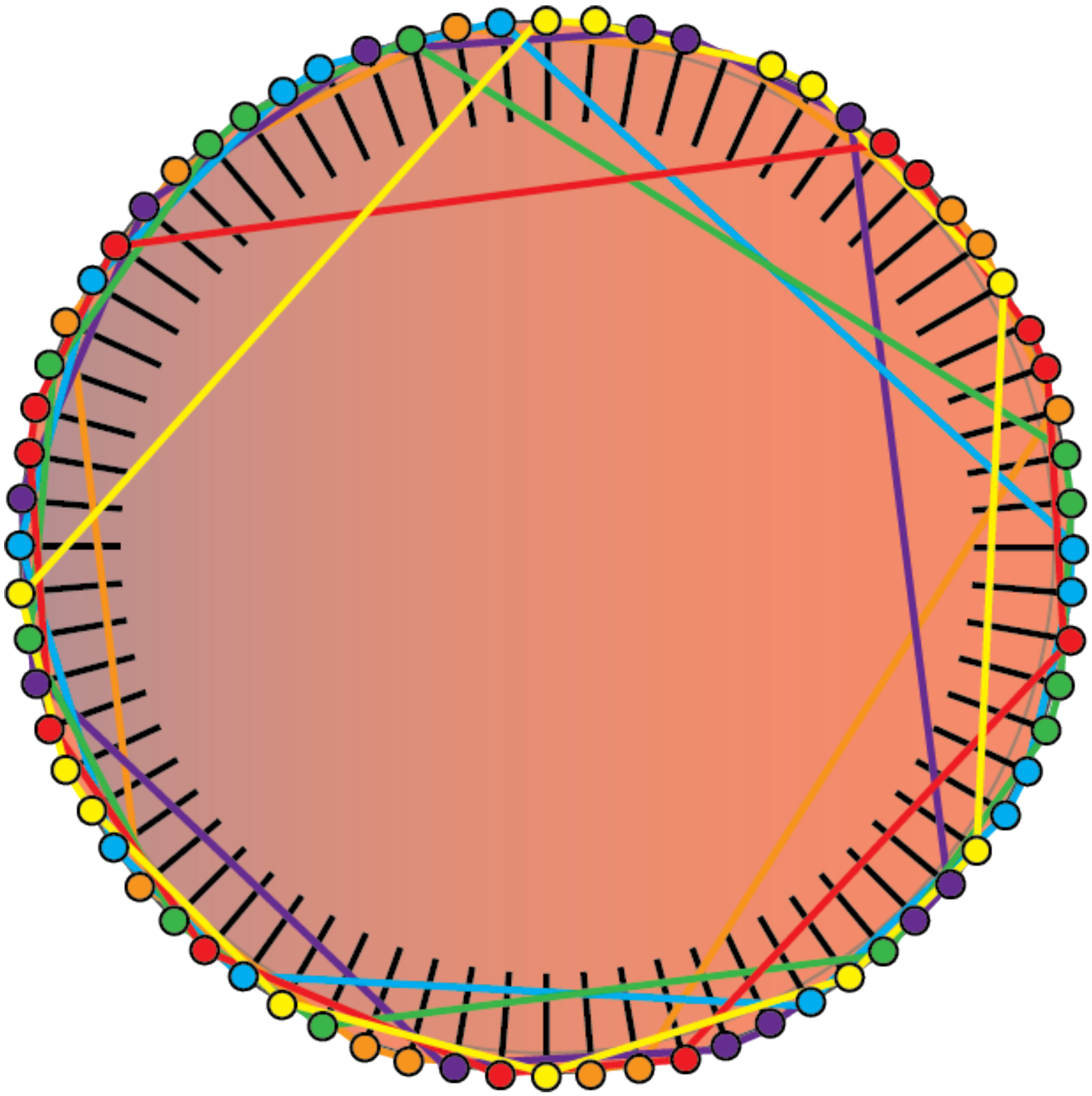

Figure 32. Smallest possible Vuza Canon of 72 pulses duration, tiled in 6 voices.

EXAMPLES OF RHYTHMIC TILING

Although mathematics needs to be employed to investigate the full extent of tiling in one dimension, there are numerous examples of rhythmic tiling of one category or another that have found a natural position in the music of traditions from around the world.

BALINESE KOTEKAN

The unique interlocking *kotekan* patterns of Balinese gamelan *gong kebyar* orchestra exhibit a seamless tapestry – the pulse train – built from an interlocking superstructure of two parts – called *polos* and *sangsih*. *Kotekan* presents a continuous stream of pulses usually performed at high speed, and *polos* and *sangsih* are typically arranged and executed in such a way as to share pitch material and a blend of on- and off-beat rhythms in a way that prioritises the resulting combination over the individuality of the parts. That is, the two parts that comprise *kotekan* perform disjunct pitch material, whose combination makes a conjunct whole.[28]

Only the *nyog cag* type of *kotekan* creates a tiling of purely disjoint motifs with non-adjacent onsets in loops. Figure 33 shows a notated example where the *polos* and *sangsih* parts are effectively complements of each other.

Figure 33. Example of *nyog cag* type of *kotekan* showing the interlocking nature of *polos* and *sangsih* (top two systems) upon a related ground bass *pokok* melody.

The other types of *kotekan* create tiling using *partially disjoint* motifs, featuring onset overlaps (but no holes or gaps in the pulse stream). *Norot* for example features the same translational symmetry of *nyog cag* but the presence of adjacent onsets results in onset overlaps. The pitches that are overlapped are part of the design and overall charm of this type of *kotekan*.

[28] See Dimond 2019 for a more in-depth analysis of the symmetries of *kotekan*.

Figure 34. Example of *norot* type of *kotekan* showing the partially disjoint motif shared by *polos* and *sangsih* (top two systems) upon a related ground bass *pokok* melody.

AFRICAN PARADIDDLE

Balinese music is not the only genre to exploit the illusory aural effects that arise from high-speed interlocking parts. The temporal features of the music of West Africa and the West African diaspora typically propagate a multiplicity in perception that subverts the primacy of the downbeat in favour of the cycle, the flow, and the relative inter-relationships of onsets. The aforementioned *paradiddle* reflection rhythm itself originates from the *balafon* (type of xylophone) of West Africa (Toussaint 208).

The ethnomusicologist Gerhard Kubik has studied the so-called *inherent rhythms* intrinsic to the instrumental music of Eastern and Central Africa, featured in the compositions for xylophone and guitar (Kubik 33).

Figure 35 notates the single paradiddle of Figure 18, whilst Figures 36 and 37 notate the double and triple paradiddles, which are longer versions of the same reflection rhythm.

Figure 35. Single paradiddle.

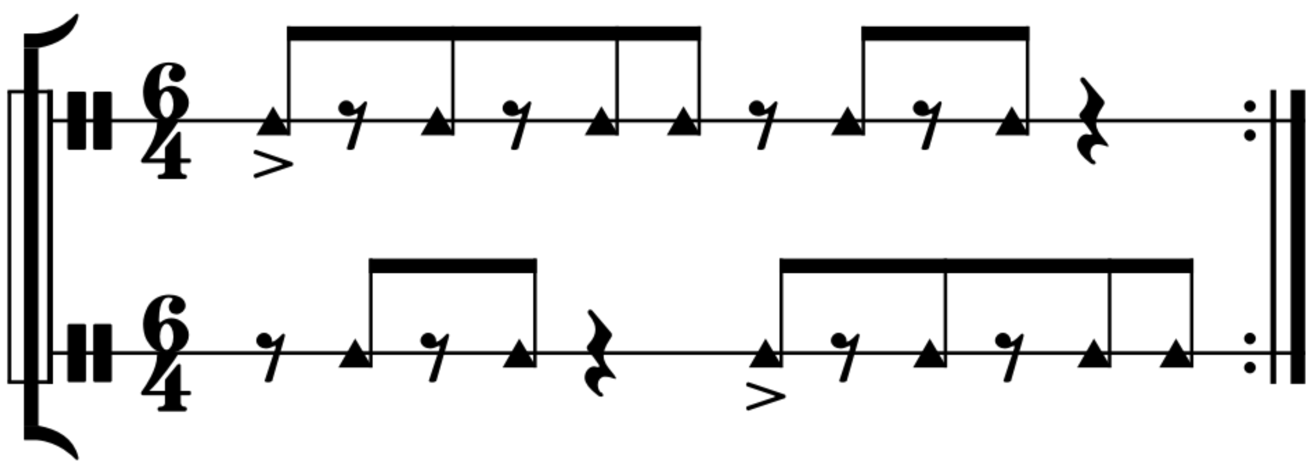

Figure 36. Double paradiddle.

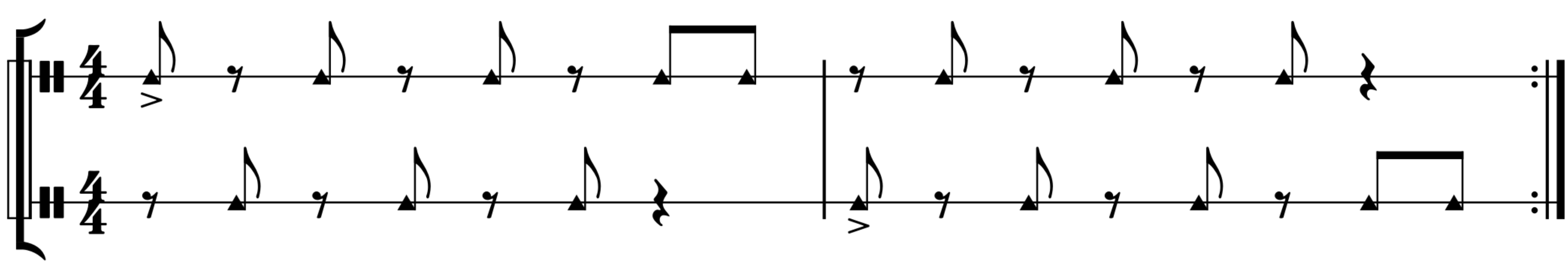

Figure 37. Triple paradiddle.

MOROCCAN GARAGAB

The performance practice of the iron castanets used in the Maghreb region of North Africa (including Algeria, Morocco, Libya and Tunisia) provide an example of interlock and pulse train as an essential element to the percussion accompaniment to voices and instruments. The *garagabs* (also known as *krakebs*) can be played as a pair by a single performer, or more often, by multiple players resulting it temporal and timbral richness. One example of a reflection rhythm played by alternating hands is illustrated in Figure 38.[29]

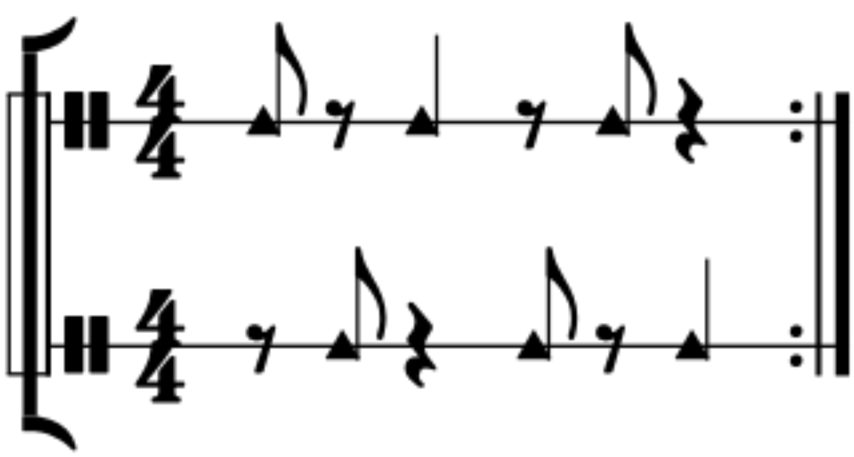

Figure 38. Moroccan *garagab* pattern.

[29] See Toussaint 212 for a discussion of the *alternating hands method* for creating reflection rhythms such as this and paradiddles.

FLAMENCO PALMAS

The patterns of hand-clapping that comprise the *palmas* of the various *compas* (metres) of Flamenco music also feature pulse trains with timbral variety (by virtue of interlocked performers and also different clapping techniques). The practice of *counter-palmas* demonstrates a direct application of complementary rhythms and non-uniform tiling, as the performer is directed to clap within the gaps of another performer's pattern. Figure 39 notates an example of palmas belonging to the *rumba* rhythm.[30] The bottom stave represents the tapping of the foot.

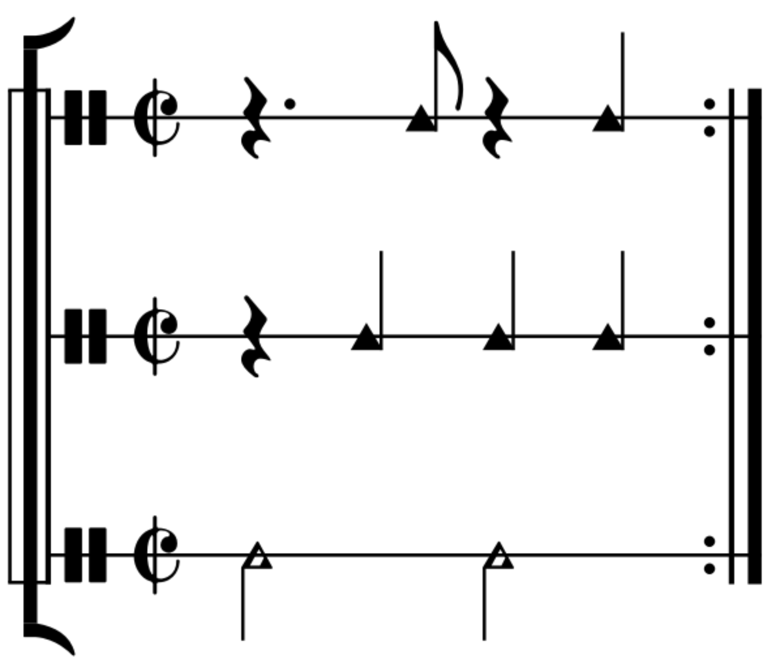

Figure 39. Flamenco *rumba* pattern.

JAZZ

The introduction to Chick Corea's "Tumba" features improvisation with a variation of the African Bell pattern, tiled between Corea's mallets on the piano and Vinnie Colaiuta's cow bell, as illustrated in Figure 40. The tiling has duplicated pulses so is partially disjoint.

[30] Rumba is one of eight families of rhythm in Flamenco, in different metres.

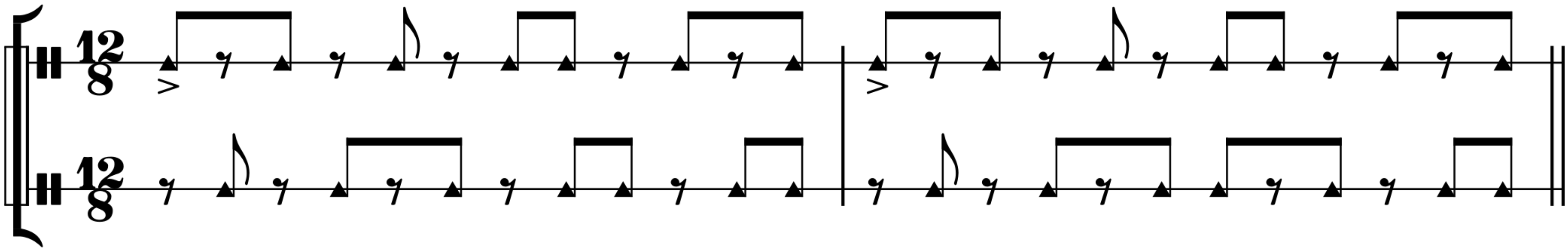

Figure 40. Pattern from Chick Corea's "Tumba".

CONCLUSION

Musical tiling is appealing to composers like myself because it contains aspects of canon, symmetry, self-similarity, and the economic usage of material. Tiling can be applied to composition in a range of ways that need not be strict and sound autonomic, but can contribute a subtle and nuanced texture to a work as well as structural cohesion.

The presence of the principles of tiling in different musical traditions demonstrate its appeal and musical value. Fascinating insights arise when one asks "why"? It is postulated that humans revel in the effects of perceptual rivalry and multiplicity in perception that arise from musical tiling, just as we do in visual art. The *double image* technique present in Salvador Dali's *Slave Market with the Disappearing Bust of Voltaire* is one example, as are the works of M.C. Escher, and the unstable perception created by the geometrical form of the Necker cube.

With tessellations of musical time, then, perhaps we enjoy the experience of the journey, and furthermore find it appealingly magical when we cannot be certain of the mode of musical transportation that carries us along?

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